



Expressions and Equations Webinar



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Presenters

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Technical Items

If you have speakers connected to your computer you should be able to hear audio through that mechanism.

If you cannot hear right now the number is:

- Toll-free: [1 877 568 4108](tel:18775684108)
- Access Code: 682-804-359
- Audio PIN: 40



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Overview

- Examine *Expressions & Equations* and *Functions* domain topics in:
 - Current Idaho State Standards
 - Common Core Standards
- Focus on investigating multiple representations as a means of building student understanding
- Links to MTI course



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Standards Names/Definitions

- The standards that have been in place for the past several years and are currently being assessed on ISAT will be referred to as the ***current Idaho State Standards***.
- The new standards (adopted in spring 2011 for implementation in fall 2013) will be referred to as the ***Common Core State Standards***



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Timeline for implementation of the Common Core State Standards & Smarter Balance Assessment



- Current Kindergarteners will never be tested on the existing Idaho state standards
- Current 7th graders will be required to pass a test of the new standards
- Can we wait to begin implementation?



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Domain Progressions in the CCCSS

K	1	2	3	4	5	6	7	8
Counting and Cardinality								
Number and Operations in Base Ten						Ratios and Proportional Relationships		
			Number and Operations – Fractions			The Number System		
Operations and Algebraic Thinking**						Expressions and Equations		
								Functions
Geometry						Geometry		
Measurement and Data*						Statistics and Probability		

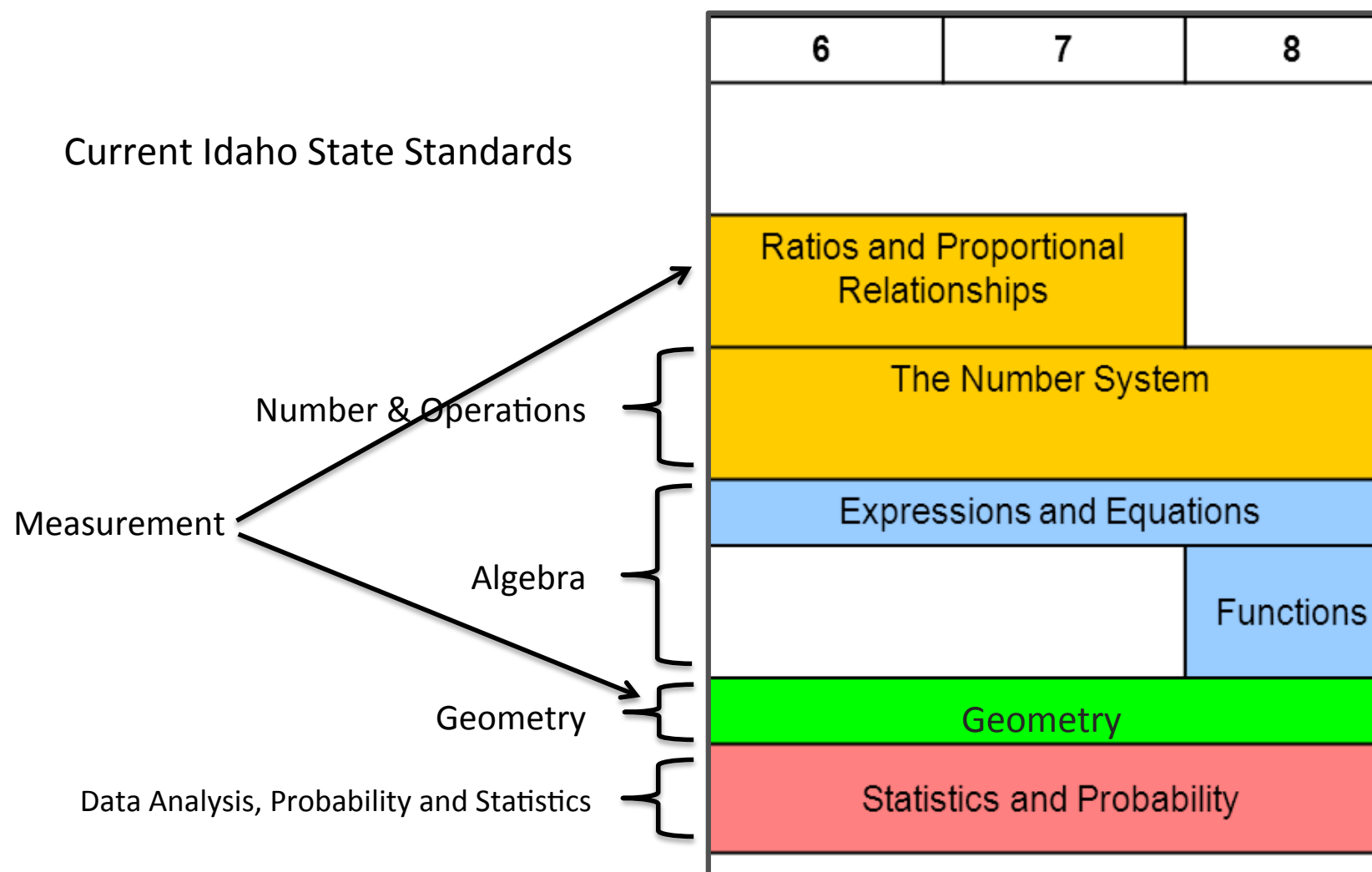
* K-5 Measurement and Data splits into Statistics and Probability and Geometry in Grade 6

**Operations and Algebraic Thinking is foundation for Grade 6 Expressions and Equations and The Number System



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Domain Progressions in the CCCSS



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Goal 3.1: Use algebraic symbolism as a tool to represent mathematical relationships.

6.M.3.1.1 Discuss the meaning and use of variables in simple expressions and equations.

6.M.3.1.2 Translate simple word statements into algebraic equations.

6.M.3.1.3 Read and use symbols of " $<$," " $>$," and " $=$ " to express relationships.

Goal 3.2: Evaluate algebraic expressions.

6.M.3.2.1 Use the following properties in evaluating numerical expressions: commutative, associative, identity, zero, inverse, and distributive.

6.M.3.2.2 Evaluate simple algebraic expressions using substitution.

Goal 3.3: Solve algebraic equations and inequalities.

6.M.3.3.1 Solve one-step equations with whole numbers.

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	Generating equivalent expressions	Solving equations (and inequalities)	Developing concept of function
6th	6 EE: Apply and extend previous understandings of arithmetic to algebraic expressions.	6 EE: Reason about and solve one-variable equations & inequalities.	6 EE: Represent and analyze quantitative relationships between dependent and independent variables.
7th	7 EE: Use properties of operations to generate equivalent expressions.	7 EE: Solve real-life and mathematical problems using numeric and algebraic expressions and equations.	7 RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.
8th	8:EE: Work with radicals and integer exponents.	8:EE: Solve linear equations in one variable.	8:EE: Understand the connections between proportional relationships, lines, and linear equations.
		8:EE: Analyze and solve pairs of simultaneous linear equations.	8:F: Define, evaluate, and compare functions. & Use functions to model relationships between quantities.

Two conceptions of variable:

Solving equations

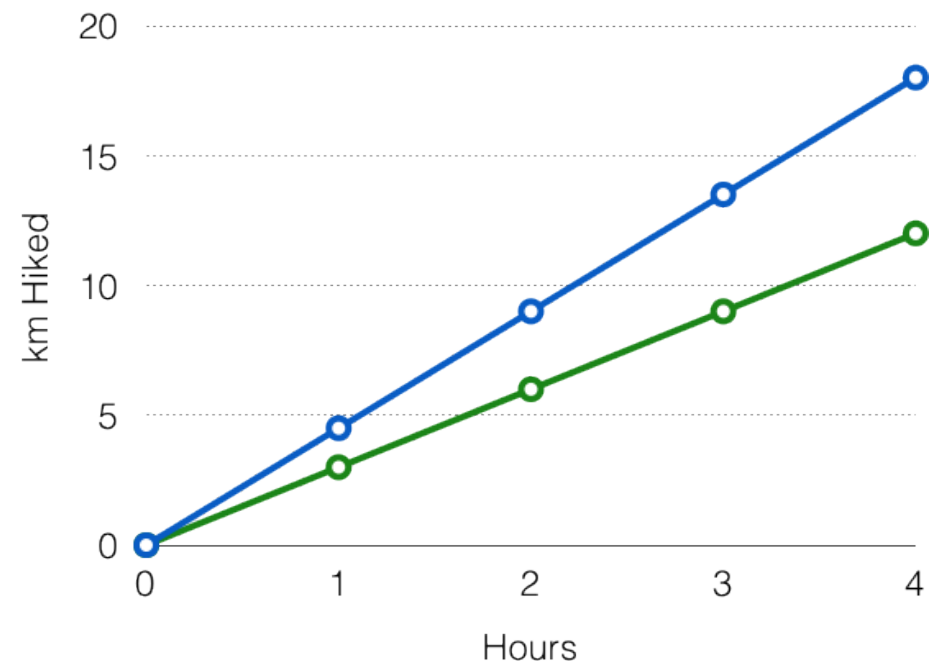
Variable as unknown

Developing concept of function

Variable as varying quantity

Two identical silver bars plus 2 ounces weigh 7 ounces all together. How much does one silver bar weigh?

$$\begin{array}{r} 2x + 2 = 7 \\ -2 \quad -2 \\ \hline 2x = 5 \\ \frac{2x}{2} = \frac{5}{2} \end{array}$$



— $d = 5t$
— $d = 3.5t$

d = km Hiked
 t = Hours Hiked

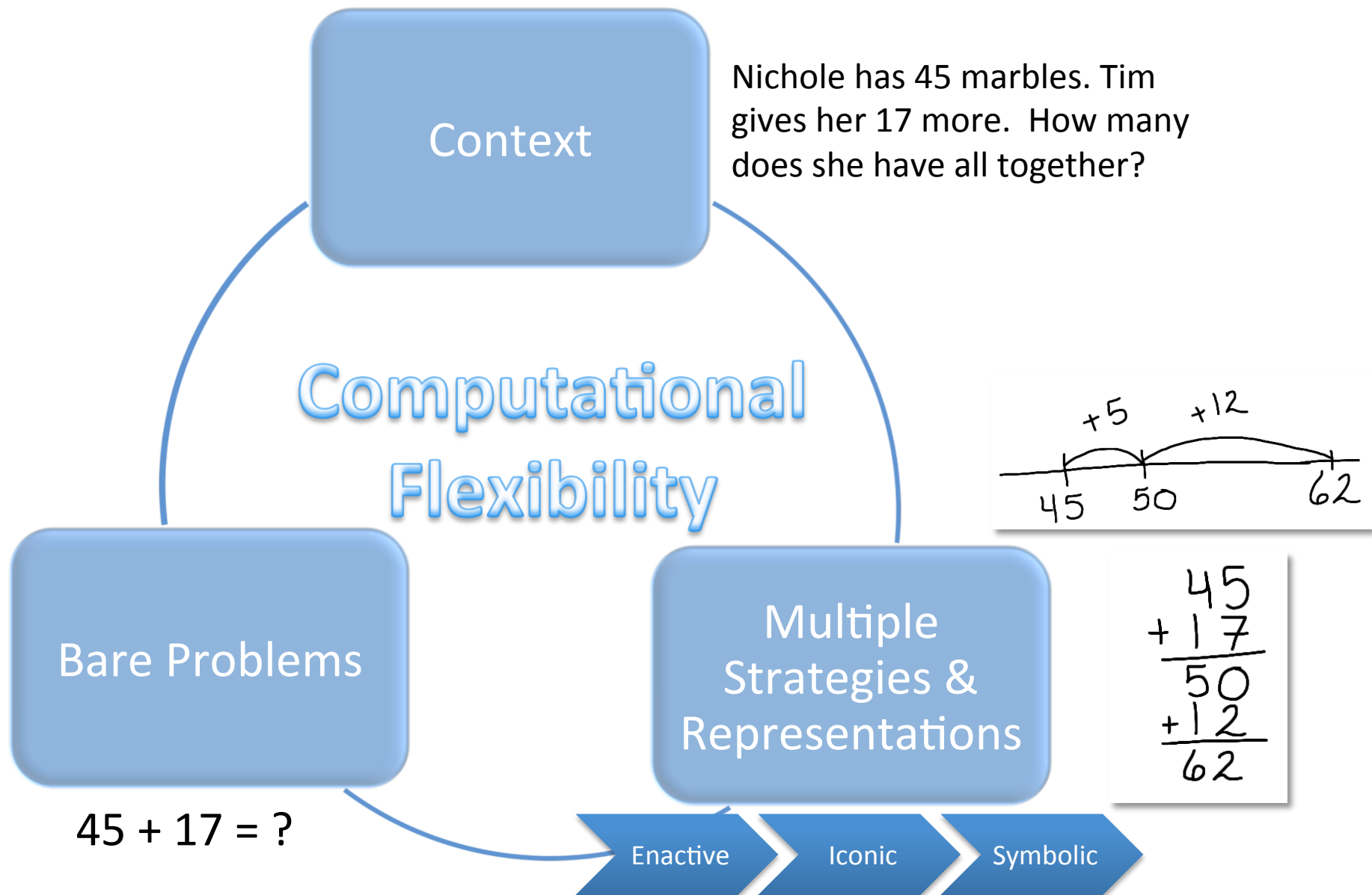
Variable as an Unknown

MULTIPLE REPRESENTATIONS AND STRATEGIES



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Context

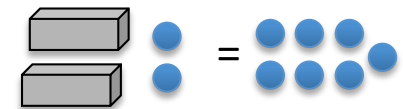
Two identical silver bars plus
2 oz weigh 7 oz all together.
How much does one silver bar
weigh?

Algebraic
Flexibility

Bare Problems

$$2b + 2 = 7$$

Multiple
Strategies &
Representations



$$\begin{array}{r} 2x + 2 = 7 \\ \hline 2 \quad 2 \end{array}$$
$$x + 1 = 3.5$$
$$x = 2.5$$

Enactive

Iconic

Symbolic



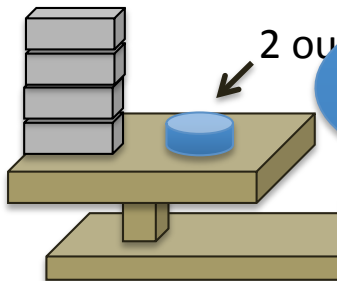
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Context

Sarah has 6 identical silver bars and is trying to figure out how much just one weighs.

She knows 4 bars plus 2 ounces weighs the same as 2 bars plus a 10 ounce weight.

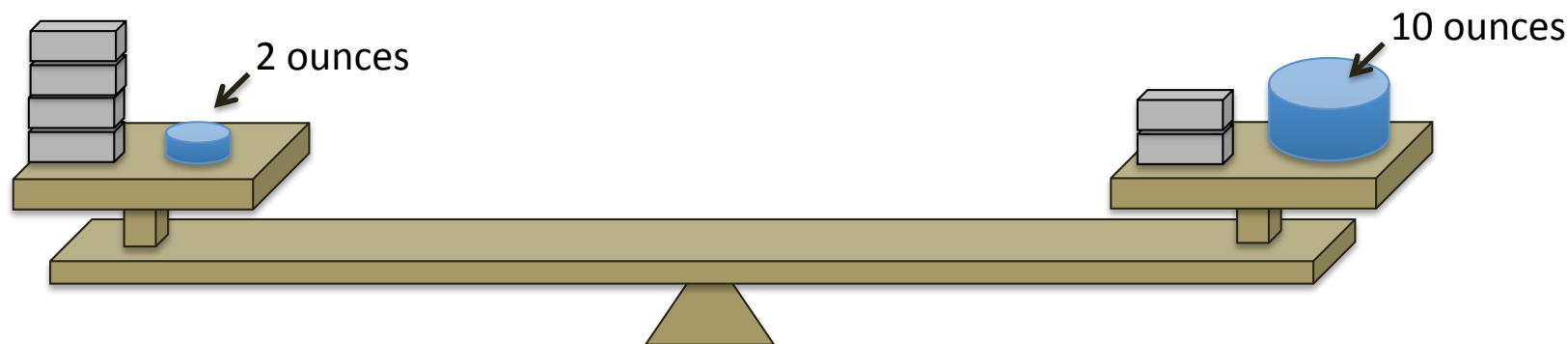
Help her figure out the weight of one bar.



Model the problem and your solution visually and with formal algebra. How might students solve the problem?



Informal Strategy: Guess and Check



If the bar weighs one ounce...

$$4(1) + 2 \stackrel{?}{=} 2(1) + 10$$
$$6 \neq 12$$

If the bar weighs two ounces...

$$4(2) + 2 \stackrel{?}{=} 2(2) + 10$$
$$10 \neq 14$$

And so on...



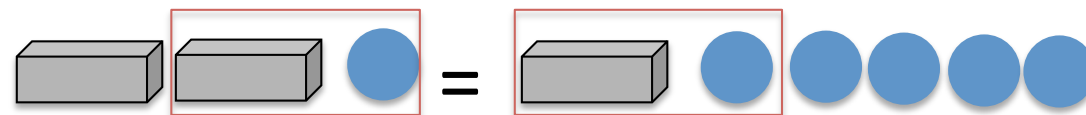
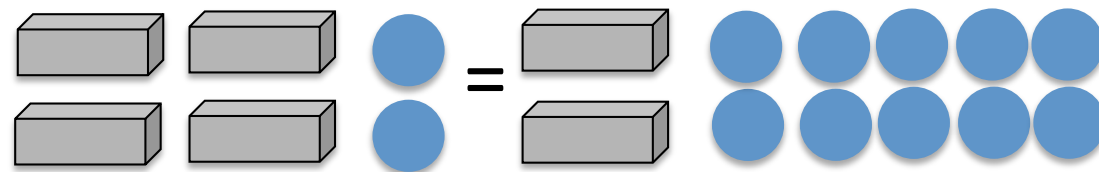
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Model: Enactive -> Iconic

 Represents 1 ounce

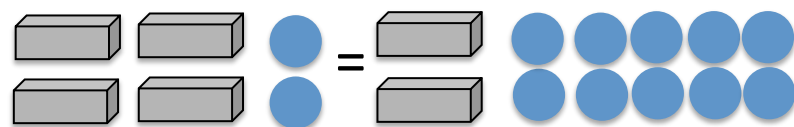
 Represents 1 silver bar

How many ounces is 1 silver bar?



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Model: Iconic -> Symbolic



$$\frac{4b + 2}{2} = \frac{2b + 10}{2}$$



$$2b + 1 = b + 5$$

$$b + \underline{b + 1} = \underline{b + 1} + 4$$



$$b = 4$$



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Model: Iconic -> Symbolic

b	b	b	b	2
b	b	10		

$$4b + 2 = 2b + 10$$

$$\quad -2 \quad \quad -2$$

b	b	b	b
b	b	8	

$$4b = 2b + 8$$

$$2b - 2b$$

b	b
8	

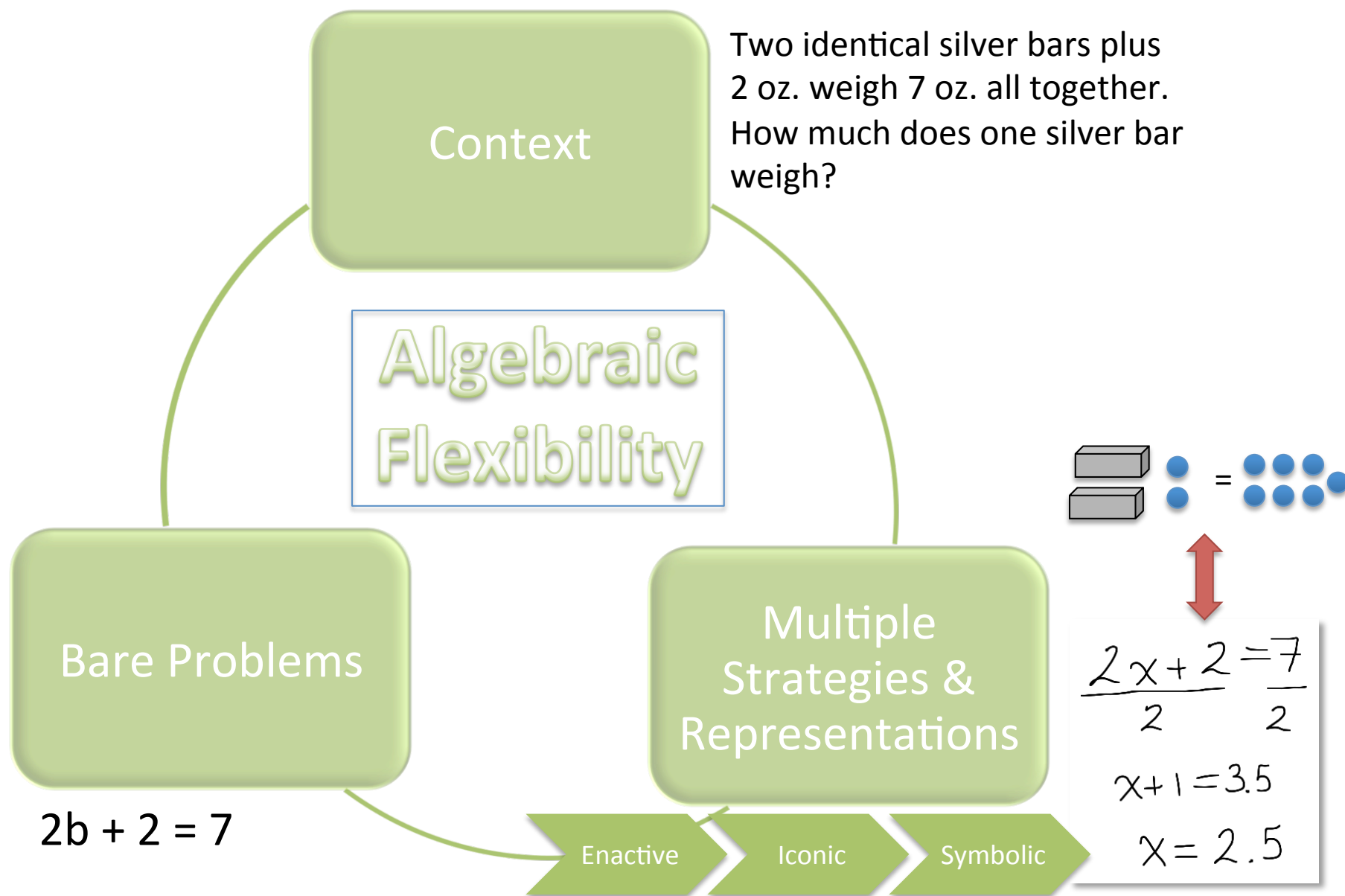
$$\frac{2b}{2} = \frac{8}{2}$$

b
4

$$b = 4$$



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Flexibility

$$\frac{1}{2}(5 + 3x) = 15$$

Algorithmic Solution

$$\frac{1}{2}(5) + \frac{1}{2}(3x) = 15$$

$$2.5 + 1.5x = 15$$

$-2.5 \quad -2.5$

$$\frac{1.5x}{1.5} = \frac{12.5}{1.5}$$

$$x = 8.\overline{3}$$

Flexible Solution

$$5 + 3x = 30$$

$-5 \quad -5$

$$\frac{3x}{3} = \frac{25}{3}$$

$$x = 8.\overline{3}$$



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Understanding Equality

$$34 + 21 = \square + 23$$

Operational versus relational understanding of the equal sign.

“A sign connecting the answer to the problem.”

“What the problem’s answer is.”

“The total.”

“How much the numbers added together equals.”

(From Knuth 2006)



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Understanding Equality

$$34 + 21 = \square + 23$$

Operational versus relational understanding of the equal sign.

“The left side of the equals sign and the right side of the equals sign are the same value.”

“The same as, the same value”

“The expression on the left side is equal to the expression on the right side.”






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Understanding Equality


$$34 + 21 = \square + 23$$

Operational versus relational understanding of the equal sign.



$$34 + 21 = \boxed{55} + 23$$


$$34 + 21 = \boxed{32} + 23$$

55 55



+32


$$34 + 21 = \square + 23$$
$$34 + 21 = \boxed{32} + 21 + 2$$

$$3x + 2 = 29$$


$$3(?) + 2 = 29$$

$$4x + 2 = 2x + 10$$



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Variable as a Varying Quantity

MULTIPLE REPRESENTATIONS



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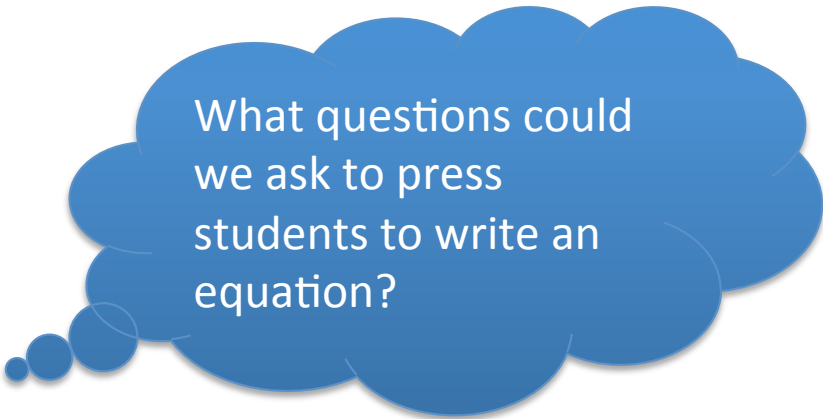
Context: Jane has \$18 and decides to save \$2 per week.

- How much money does she have after 2 weeks? 10? 15?
- How long will it take her to get to \$63?

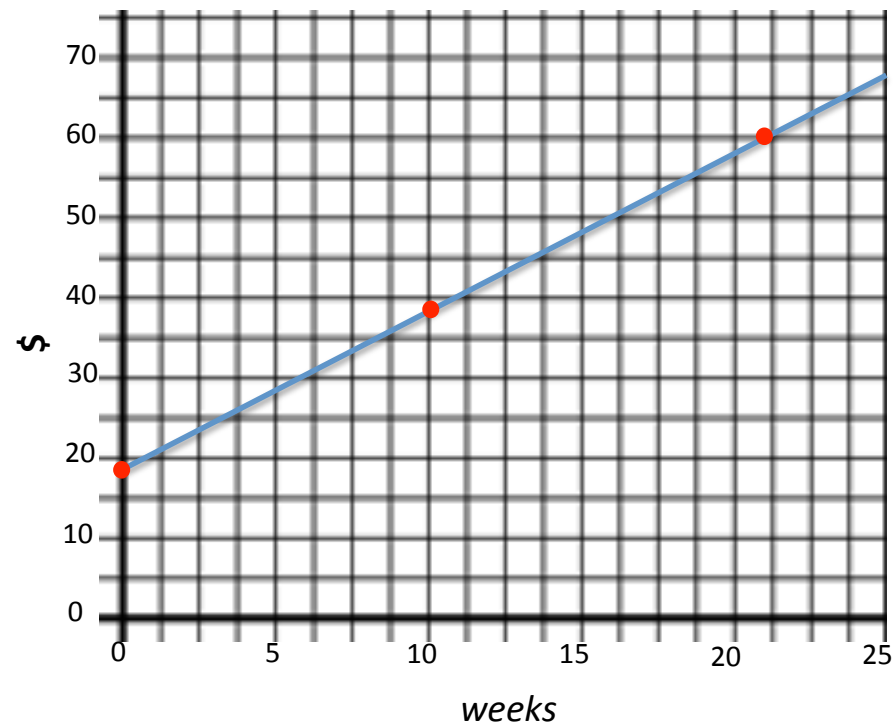
Table: Create a table to solve the problem or display your answers.

\$	18	20	22	26	30	34	38	58	48	60	62	64
wks	0	1	2	4	6	8	10	20	15	21	22	23

Equation: ??



Graph: Create a graph to solve the problem or display your answers.



Context: Jane has \$18 and decides to save \$2 per week.

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- How long will it take her to get to \$63?

Table: Create a table to solve the problem or display your answers.

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Equation: Write equations that would help answer the following questions:

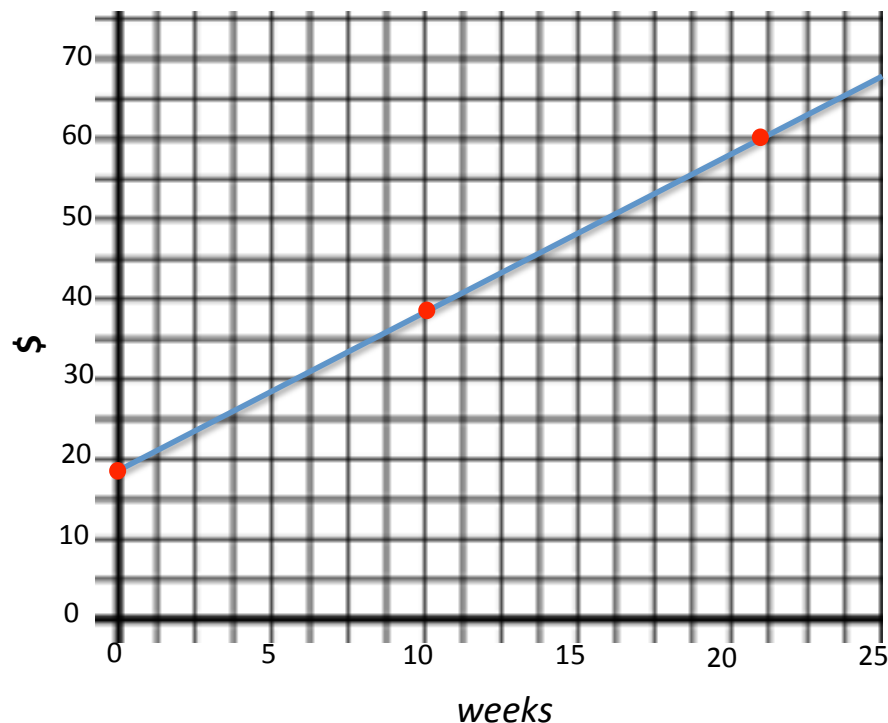
- How much money does she have if you know how many weeks have passed?
- How many weeks have passed if you know how much money is in her account?

$$\text{\$} = 18 + 2 \cdot \text{wks}$$

$$\text{wks} = \frac{\text{\$} - 18}{2}$$

$$y = 2x + 18$$

Graph: Create a graph to solve the problem or display your answers.



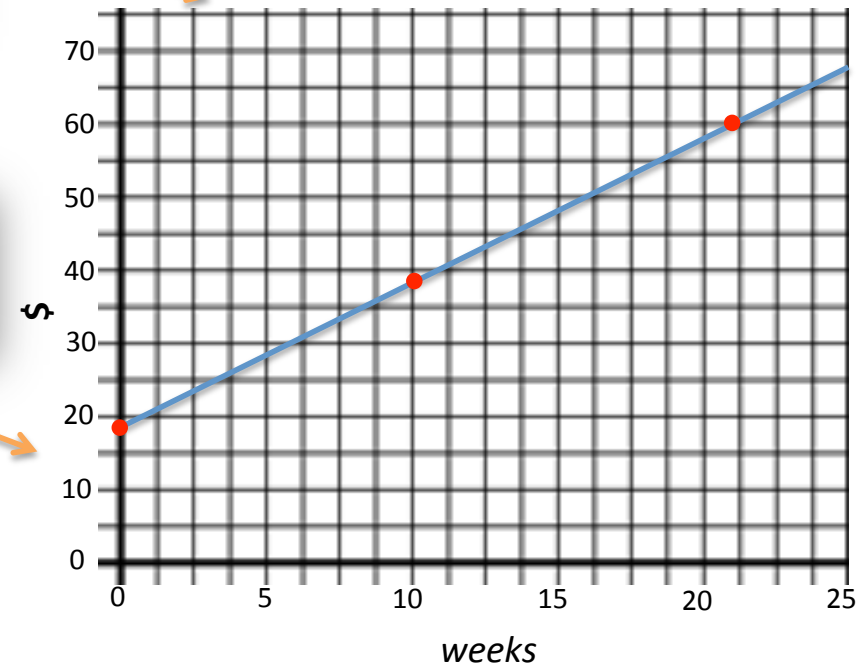
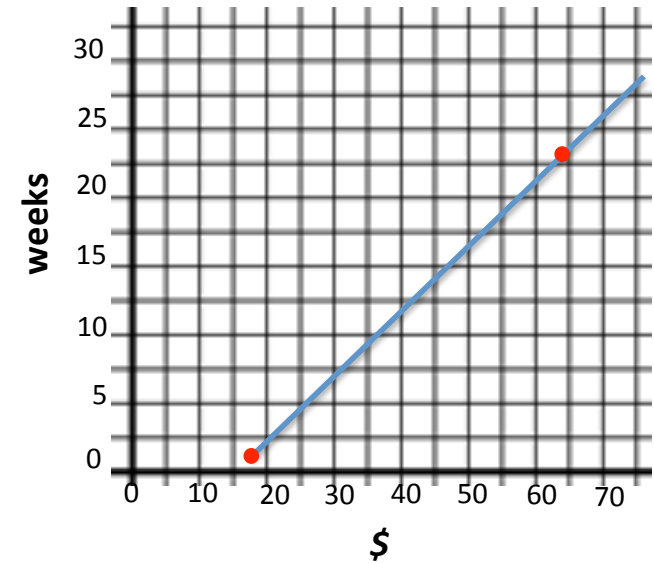
Are all the representations correct? What connections can you see between them?

\$	18	20	22	26	30	34	38	58	48	60	62	64
wks	0	1	2	4	6	8	10	20	15	21	22	23

$$\text{\$} = 18 + 2 \cdot \text{wks}$$

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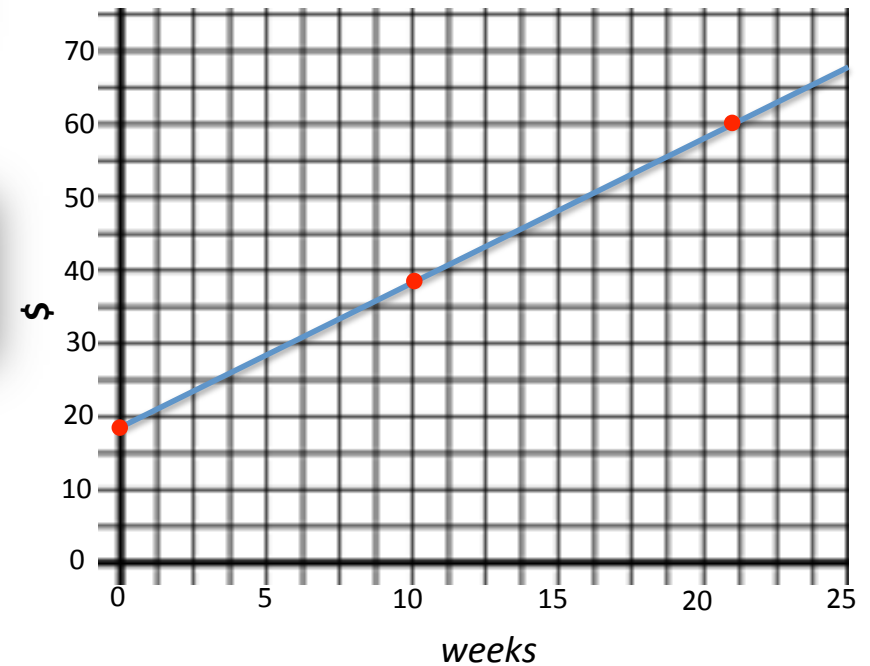
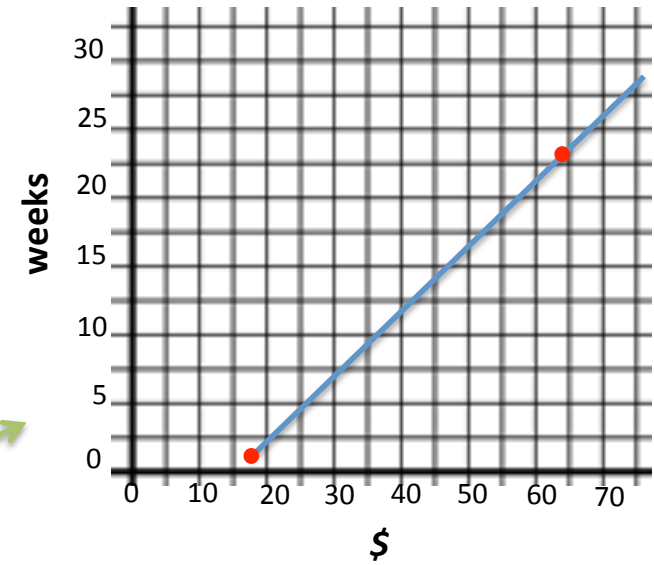


\$	18	20	22	26	30	34	38	58	48	60	62	64
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$$\text{\$} = 18 + 2 \cdot \text{wks}$$

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Context

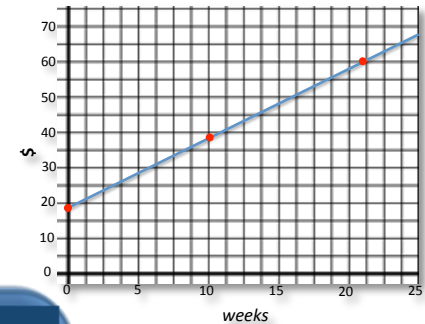
Jane has \$18 and decides to save \$2 per week.

Relationship Perspectives

Bare Problems

$$y = 18 + 2x$$

Multiple Representations

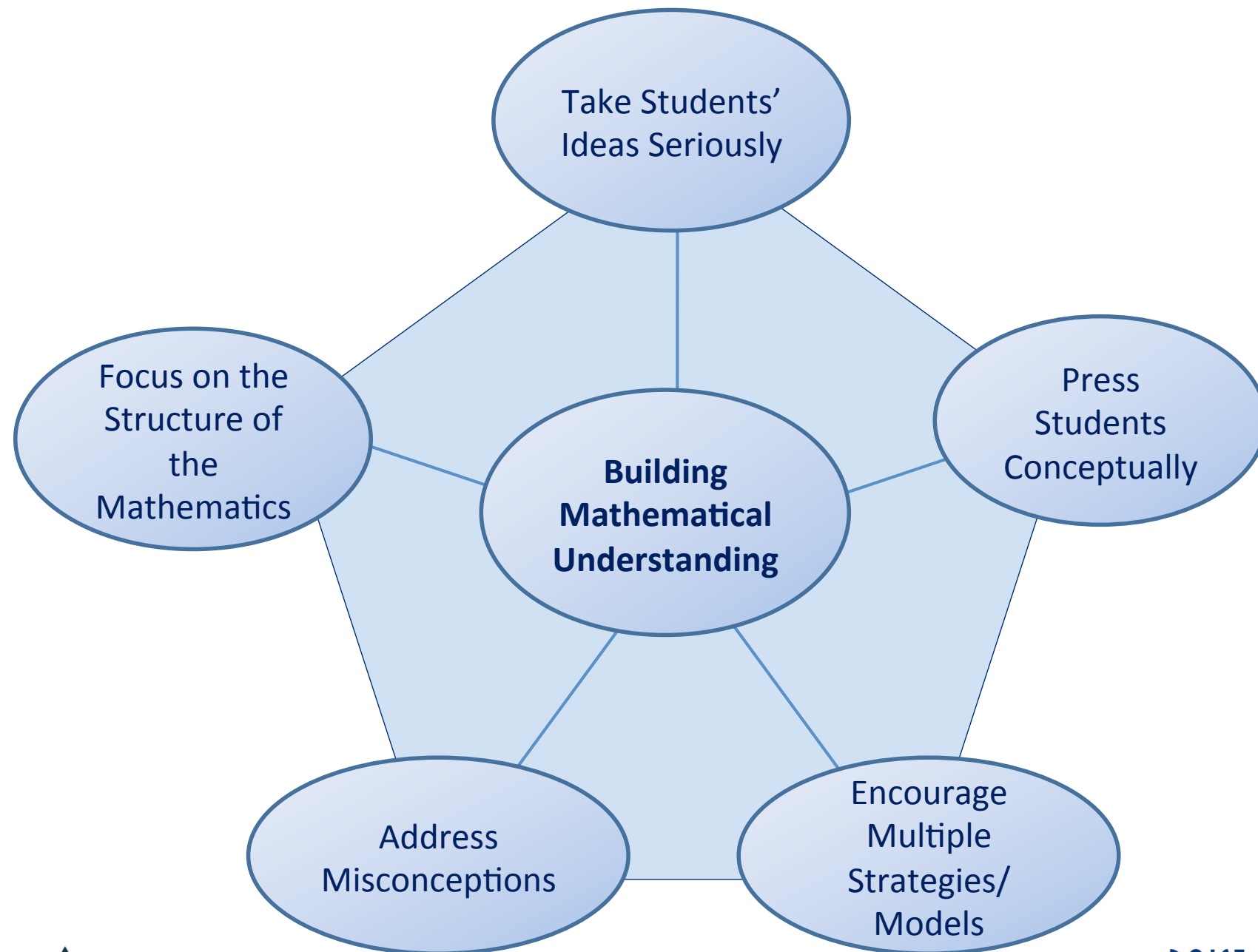


$$\text{\$} = 18 + 2 \cdot \text{wks}$$

\$	18	20	22	26	30
wks	0	1	2	4	6



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1 Credit Opportunity

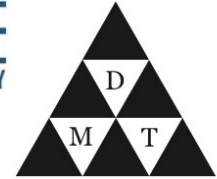
- Duration: Accumulate 15 hours of webinar training, live or archived. Additional webinars will be developed and offered during the Fall of 2012. The credit will be earned the semester the 15 hours is completed.
- Registration: Upon completion of the 15 hours, a participant will register with BSU for the one professional education credit.
- Documentation: Completion of a brief webinar summary and reflection for each webinar is required.
- Cost: \$65
- Note: The one professional education credit earned for completion and payment of \$65, does not count towards the three credits earned with completion of the MTI course. The webinars are follow-up support after completion of the MTI course.
- Information: <http://www.sde.idaho.gov/site/math/mti.htm>
- Questions: Nichole Hall nhall@sde.idaho.gov



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Thank you for attending the webinar!

- Questions
- Contact Information
 - Michele Carney michelecarney@boisestate.edu
 - Gwyneth Hughes gwynethhughes@boisestate.edu
- DMT Website- <http://dmt.boisestate.edu>
- Follow Up Opportunities:
<http://www.tinyurl.com/mtifollowup>



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